

Rotating black holes at future colliders II: Anisotropic scalar field emission

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Abstract

This is the sequel to the first paper of the series, where we have discussed the Hawking radiation from five-dimensional rotating black holes for spin 0, 1/2 and 1 brane fields in the low frequency regime. We consider the emission of a brane localized scalar field from rotating black holes in general space-time dimensions without relying on the low frequency expansions.

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I. INTRODUCTION

Black hole is one of the most important key objects in theoretical physics. Though its quantum behavior and thermodynamic property have played great roles in the path to understand yet unknown quantum theory of gravity (see e.g. Refs. [1, 2, 3]), a direct experimental test had been believed almost impossible. Recently, the scenarios of large [4] and warped [5] extra dimension(s) have led to an amazing possibility of producing black holes at future colliders with distinct signals [6, 7] (see also Refs. [8, 9] for studies before the observation [10] that black holes radiate mainly on the brane).

When the center-of-mass (CM) energy of a collision exceeds the Planck scale, which is of the order of TeV here, the cross section is dominated by a black hole production [11], which is predicted to be of the order of the geometrical one [12, 13, 14, 15], increasing with the CM energy. In this trans-Planckian energy domain, the larger the CM energy is, the larger the mass of the resulting black hole is, and hence the better its decay is treated semi-classically via Hawking radiation [16]. Main purpose of this series of work is to discuss such decay signals in the hope that these will serve as the basis to pursue stringy or quantum gravitational corrections to them.

In previous publications, we have pointed out that the production cross section of a black hole increases with its angular momentum, so that the produced black holes are highly rotating [17, 18] (see also Ref. [19] for an earlier attempt). The form factor for the production cross section, taking this rotation into account [17], is larger than unity and increases with the number of dimensions $D = 4 + n$. The result is in good agreement with an independent numerical simulation of a classical gravitational collision of two massless point particles [14]. We note that this form factor is hardly interpretative without considering the angular momentum. It is indispensable to take into account the angular momentum of the black hole when we perform a realistic calculation of its production and evaporation.

Black holes radiate mainly into the standard model fields that are localized on the brane [10]. In the previous paper [17], we have shown that the massless brane field equations with spin zero, one-half and one, i.e. for all the standard model fields, are of variables separable type in the rotating black hole background. Then we obtained the analytic expressions for the greybody factor of $D = 5$ (Randall-Sundrum 1) black hole by solving the master equation under the low energy approximation of radiating field. In this work we present a

generalized result to higher dimensional black hole in $D \geq 5$ for brane localized scalar field without relying on the low energy approximation and discuss its physical implications.

This paper is the longer version of the brief report [18]. We also note that the related works by Harris [22] and by Harris and Kanti [23] appeared more recently.

II. BRANE SCALAR FIELD EMISSION

A brane-localized scalar field Φ in the higher $(4+n)$ -dimensional rotating black hole background [20] can be decomposed into the radial and angular parts $R(r)$ and $S_{\ell m}(\vartheta)$, respectively [17]

$$\Phi = R(r)S_{\ell m}(\vartheta)e^{-i\omega t+im\varphi}, \quad (1)$$

where the Boyer-Lindquist coordinate $(t, r, \vartheta, \varphi)$ reduces to the spherical coordinate at spatial infinity and ℓ, m are the angular quantum numbers. The resultant equations are shown to be separable [17]. The angular part $S_{\ell m}$ obeys the equation for the spheroidal harmonics while the radial equation becomes

$$\left[\frac{d}{dr} \Delta \frac{d}{dr} + \frac{[(r^2 + a^2)\omega - ma]^2}{\Delta} + 2ma\omega - a^2\omega^2 - A \right] R = 0. \quad (2)$$

where

$$\Delta(r) = (r^2 + a^2) - (r_h^2 + a^2) \left(\frac{r}{r_h} \right)^{1-n}, \quad (3)$$

with r_h and a being the horizon radius and the rotation parameter of the black hole, respectively. Note that $\Delta(r_h) = 0$.

The power spectrum of the Hawking radiation is governed by, for each scalar mode,

$$\frac{dE}{dt d\omega d\cos\vartheta} = \frac{\omega\Gamma_{\ell m}}{e^{(\omega-m\Omega)/T} - 1} |S_{\ell m}(\vartheta)|^2, \quad (4)$$

where Ω and T are the angular velocity and the Hawking temperature of black hole

$$\Omega = \frac{a_*}{(1 + a_*^2)r_h}, \quad T = \frac{(n+1) + (n-1)a_*^2}{4\pi(1 + a_*^2)r_h}, \quad (5)$$

with $a_* = a/r_h$. The $\Gamma_{\ell m}$ is the greybody factor that determines the departure from black body spectrum, which is the main object of this paper. One immediate observation is that the contribution from $m > 0$ modes dominates over that from $m < 0$ modes in rapidly rotating case: $\Omega \gg \omega$.

III. NUMERICAL EVALUATION OF GREYBODY FACTOR

The asymptotic forms of the radial wave function at the near horizon (NH) and far field (FF) limits, $r \rightarrow r_h$ and $r \rightarrow \infty$ respectively, are [17]

$$R_{\text{NH}} = Y_{\text{in}} e^{-ikr_*} + Y_{\text{out}} e^{ikr_*}, \quad (6)$$

$$R_{\text{FF}} = Z_{\text{in}} \frac{e^{-i\omega r_*}}{r} + Z_{\text{out}} \frac{e^{i\omega r_*}}{r}, \quad (7)$$

where $k = \omega - ma/(r_h^2 + a^2)$ and the tortoise coordinate r_* is defined by $r_*(r) \rightarrow r$ for $r \rightarrow \infty$ and

$$\frac{dr_*}{dr} = \frac{r^2 + a^2}{\Delta(r)}. \quad (8)$$

We obtain the greybody factors in the following steps.

1. Put the purely ingoing boundary condition $Y_{\text{out}} = 0$ by imposing

$$R(r_0) \rightarrow e^{-ikr_*(r_0)}, \quad R'(r_0) \rightarrow -ik \frac{r_0^2 + a^2}{\Delta(r_0)} e^{-ikr_*(r_0)}, \quad (9)$$

at the NH region $r_0 = r_h(1 + \epsilon)$.

2. Numerically integrate the master equation (2) from the NH region to FF regime $r = r_{\text{max}}$.
3. Perform a least squares fit to the obtained data by the function (7) around $r = r_{\text{max}}$ to get Z_{in} and Z_{out} .
4. Finally greybody factor for the (ℓ, m) mode is given by the absorption rate

$$\Gamma_{\ell m} = 1 - \left| \frac{Z_{\text{out}}}{Z_{\text{in}}} \right|^2. \quad (10)$$

For the angular eigenvalue A , we employed the small $a\omega$ expansions up to 6th order in [21]. (The last 6th order term in the expansion is less than a few percent of the leading order term at $a\omega \lesssim 3$ for the $(\ell, m) = (0, 0)$ and $(2, 0)$ modes and at $a\omega \lesssim 4$ for all the other modes.) We have also performed above procedure in the ingoing Kerr-Newman coordinate as a cross-check.

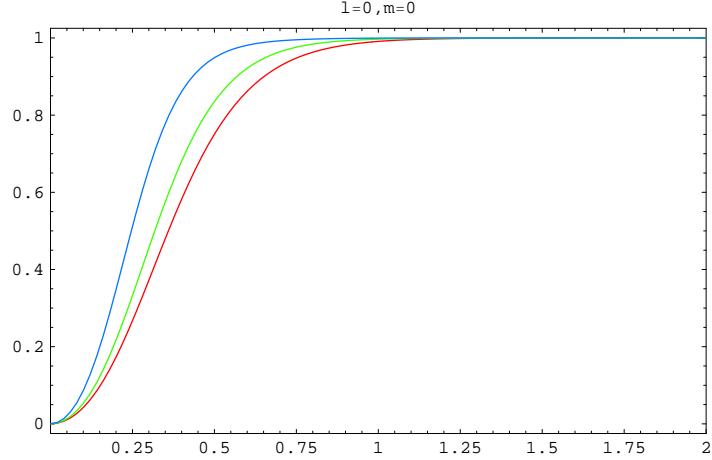


FIG. 1: Greybody factor for five dimensional black hole for the brane scalar emission into the $\ell = 0$ mode. The red, green and blue curves correspond to $a_* = 0, 0.5$ and 1.0 , respectively.

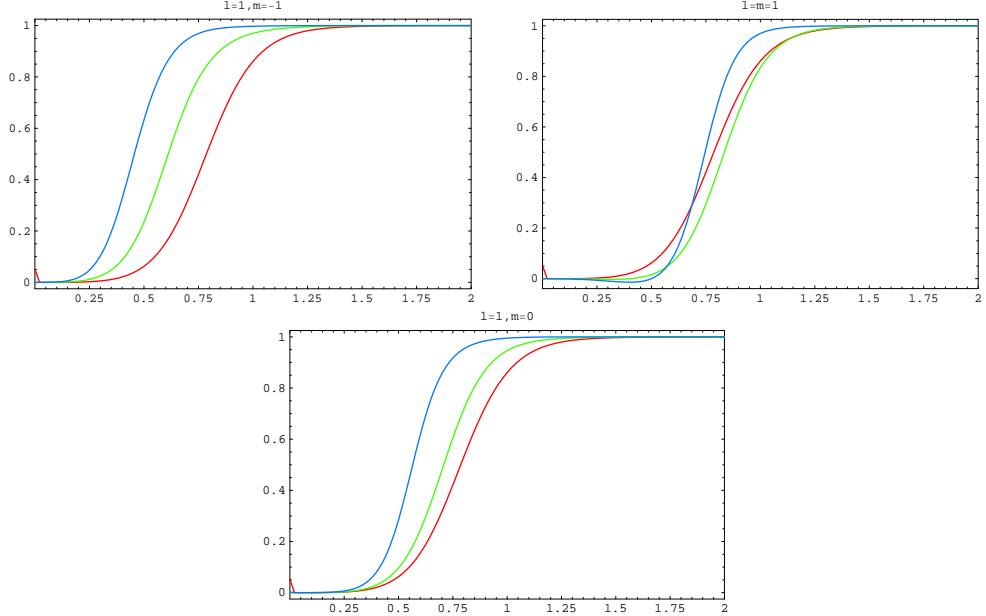


FIG. 2: Greybody factors for the $D = 4 + n = 5$ black hole for the brane scalar emission into the $\ell = 1$ modes with $m = -1, 1$ and 0 for the upper-left, upper-right and lower graphs, respectively. The $m = 1$ mode shows the superradiance, namely a negative greybody factor, in the low-frequency region $\omega < ma/(r_h^2 + a^2)$. The red, green and blue curves correspond to $a_* = 0, 0.5$ and 1.0 .

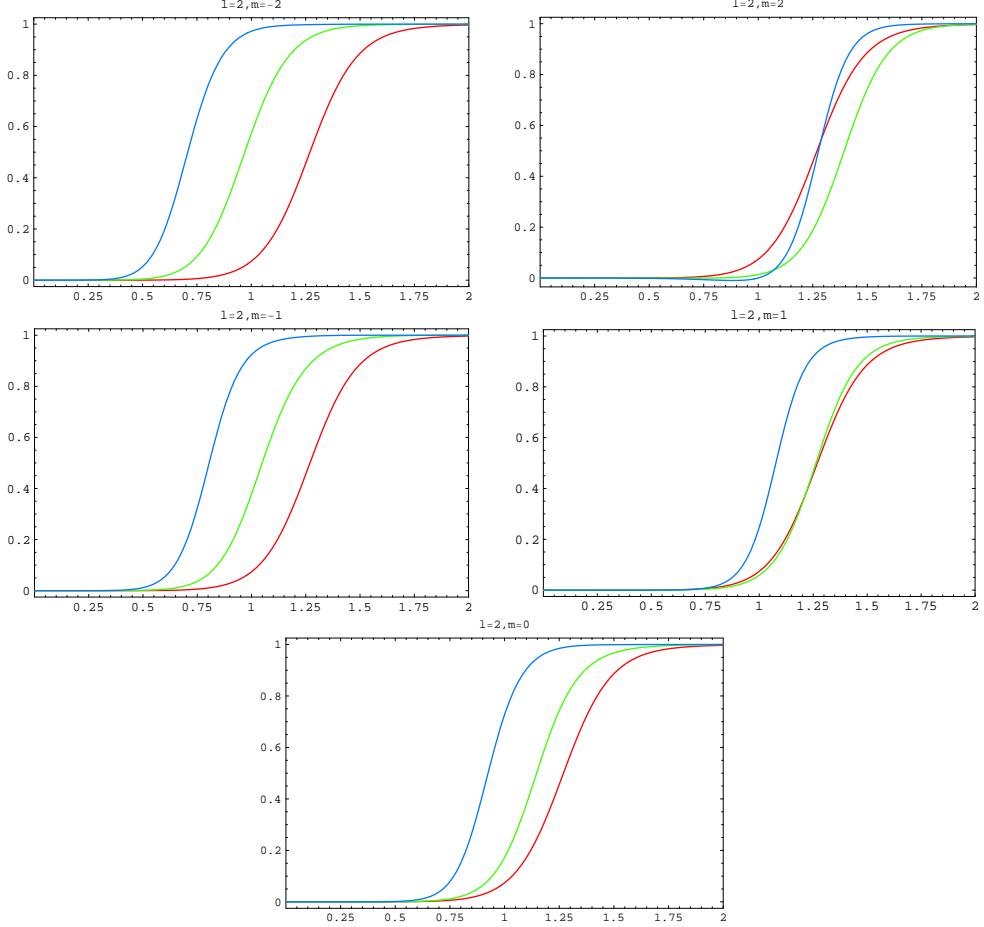


FIG. 3: Greybody factor for the $D = 4 + n = 5$ black hole for the brane scalar emission into the $\ell = 2$ modes with $m = -2, 2$ for the upper-left, upper-right, $m = -1, 1$ for the middle-left, middle-right, and $m = 0$ for the lower graphs, respectively. The $m > 0$ mode shows the superradiance, namely a negative greybody factor, in the low-frequency region $\omega < ma/(r_h^2 + a^2)$. The red, green and blue curves correspond to $a_* = 0, 0.5$ and 1.0 .

IV. RESULTS

A. Comparison with analytic expansions for $D = 4 + n = 5$

The numerical results of the greybody factors for the $D = 5$ black hole is shown in Figs. 1-3. They are in good agreement with the previous analytic expression in [17] in the

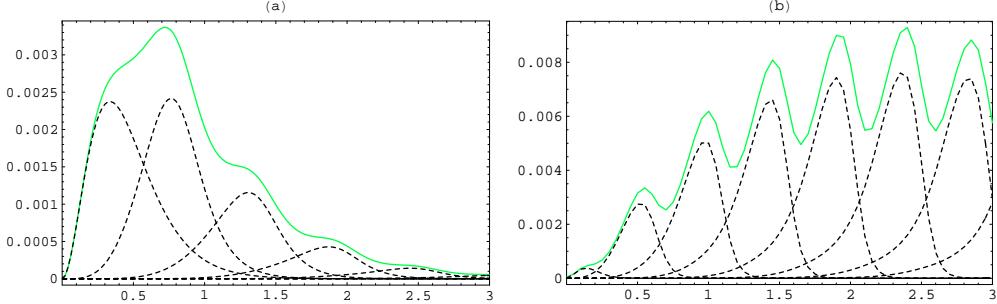


FIG. 4: The total power spectrum of five dimensional black hole with $a_* = 0.8$ and 1.5 . In Fig. (a), The green curve denotes the total power spectrum. Dotted curves correspond to $\ell = m = 0, 1, 2, 3, 4$ and 5 modes, respectively. Figure (b) shows that when the hole is rotating the total power spectrum is essentially determined by $\ell = m$ modes, with each peak corresponding to each angular mode (see also Fig. 5).

region $r_h\omega \lesssim 0.3$. At $\omega = \omega_0$ with

$$\omega_0 = m\Omega = \frac{ma}{r_h^2 + a^2}, \quad (11)$$

the Bose statics factor diverges, but this divergence is regularized to give a finite emission rate due to the zero absorption rate at this point, where greybody factor cross the zero.[26] As we claimed in Ref. [18], our analytic expression in Ref. [17] correctly shows for which ω there emerges the super-radiance with the negative greybody factor $\propto \tilde{Q} = (1 + a_*^2)\omega - ma_* < 0$.

B. Power spectrum

In Fig. 4 we show the power spectrum for $D = 4 + n = 5$ black hole. This shows that the spectrum is dominated by the $\ell = m$ modes. The result show that the domination of the $\ell = m$ mode is more significant for highly rotating case. As a check we plot the $\ell = m$ contribution and that from the other modes in Fig. 5.

Fig. 6 we plot using only the $\ell = m$ modes. We take the angular modes from $\ell = m = 0$ to 7. In the low energy region ($r_h\omega \lesssim 0.3$) the spectrum of the highly rotating black hole is suppressed compared with the non-rotating power spectrum, confirming the analytic result in low energy expansions [17], while in the higher energy regime the spectrum is greatly enhanced. This tendency is stronger for larger dimensions. To see that more explicitly, we

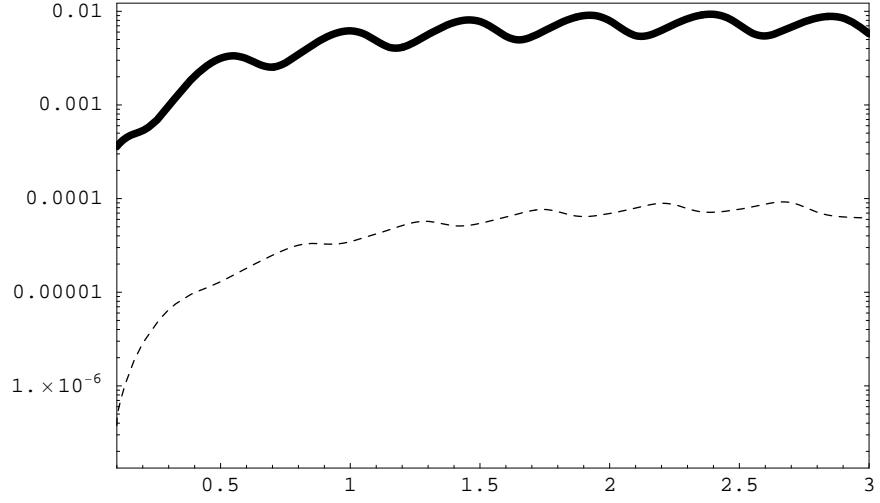


FIG. 5: Total power spectrum of $D = 4 + n = 5$ black hole with $a_* = 1.5$. The small concave curve denote the contribution from the sum of other than $\ell = m$ modes. We can safely neglect the other modes than $\ell = m$ modes.

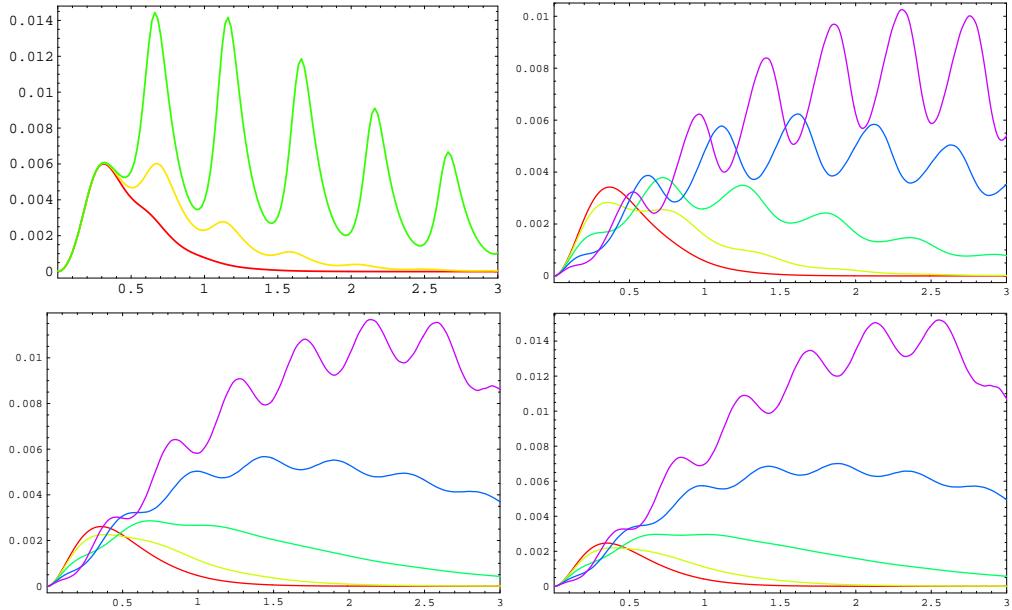


FIG. 6: Power spectrum of a rotating black hole in $D = 4 + n = 4, 5, 7, 10$ for the upper-left, upper-right, lower-left and lower-right plots, respectively. The red, yellow, green, blue and purple curves denote $a_* = 0, 0.4, 0.8, 1.2$ and 1.6 respectively. Each peak corresponds to each angular mode with $\ell = m$, see Fig. 4.

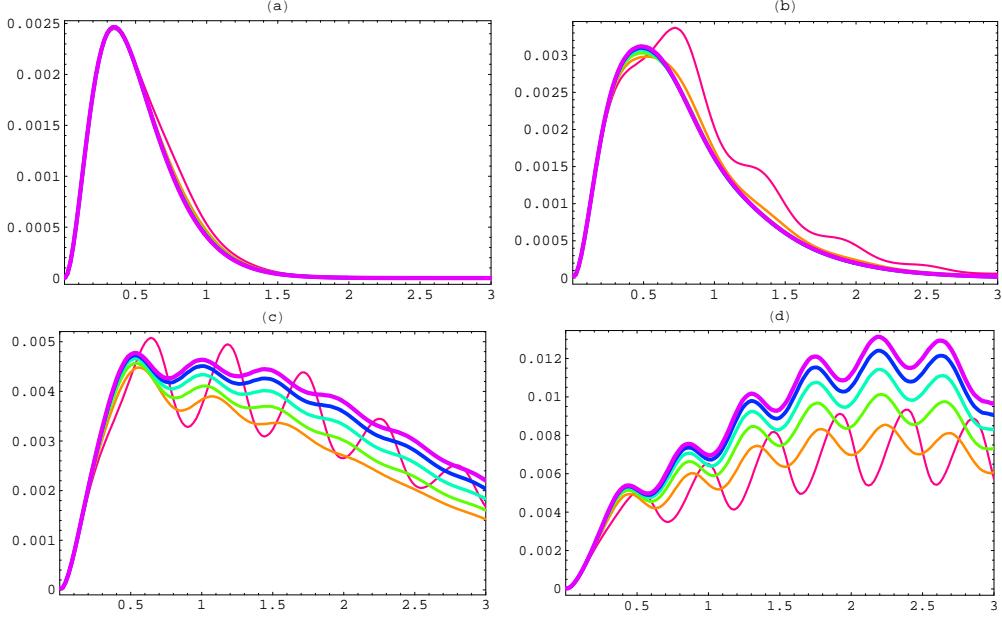


FIG. 7: Total power spectrum of higher dimensional black holes with $a_* = 0, 0.5, 1.0$ and 1.5 for the upper-left, upper-right, lower-left, lower-right plots, respectively. The pink, red, yellow, green, cyan and blue curves correspond to $D = 4 + n = 5, 6, 7, 8, 9$ and 10 , respectively.

also plot the power spectrum varying the number of dimensions in Fig. 7.

C. Angular distribution

In Fig. 8 we plot the angular distribution for $D = 5$ black holes. We approximate the spheroidal harmonics by the spherical harmonics with the assumption $a\omega \ll 1$. We confirm the previous result in the low frequency approximation [17] that the anisotropy is greatly enhanced for a highly rotating black hole, due to the $\ell = m > 0$ mode.

V. DISCUSSION

We have explained the importance of the angular momentum when one considers TeV scale black hole production and evaporation. New numerical results are shown: greybody factor for the brane scalar emission from a general $D = 4 + n$ dimensional rotating black hole without relying on the low frequency expansions. The greybody factors are obtained for general $D \geq 4$ dimensional cases and the various angular modes (ℓ, m) . We confirmed the

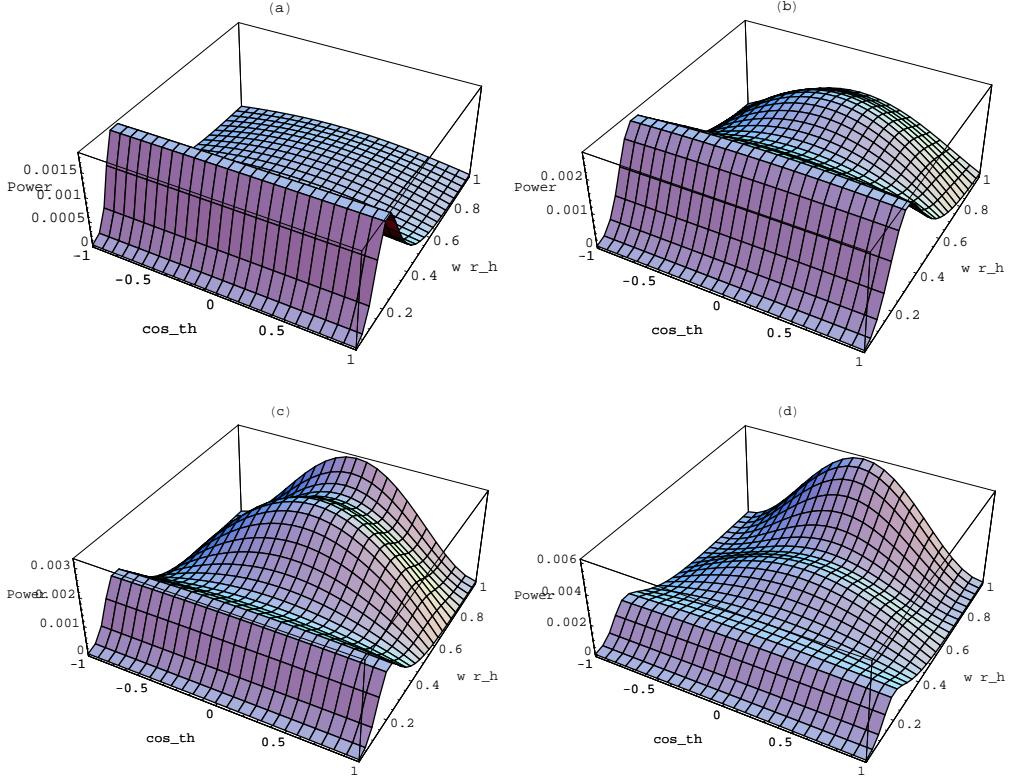


FIG. 8: Angular distribution for $D = 5$ black hole. The $a_* = 0, 0.5, 1.0, 1.5$ from Fig. (a) to (d).

nontrivial angular dependence of the scalar emission at the middle energy region $r_h\omega \sim 0.5$ and found that it is even more enhanced at higher energy region.

To understand the actual evolution of a black hole and to predict the collider signature, we need further investigations. It is important to determine the greybody factors for spinor and vector fields. The evolution of the angular momentum and the mass can be determined once all the greybody factors are determined [24, 25]. In particular, the spin-down phase, whose time evolution has been impossible to determine so far, can be precisely described. One can in principle determine the angular momentum of the produced black hole from the nontrivial angular distribution of the signals.

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